

Force and Acceleration on an Airtrack

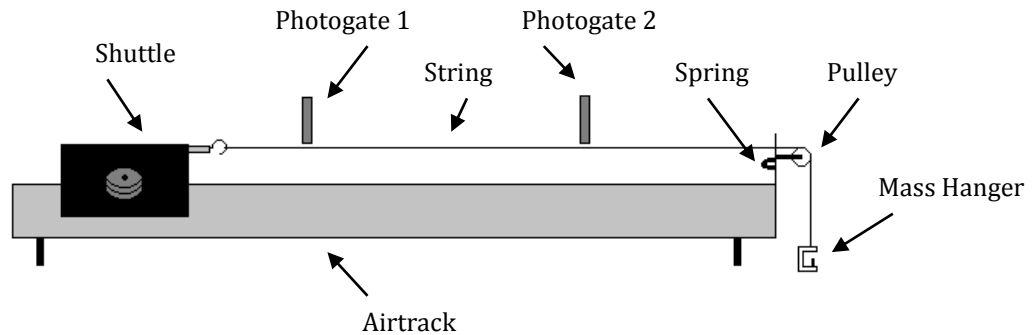
Objectives:

- Experimental objective
 - Students will verify Newton's second law of motion.
- Learning objectives (students should learn...)
 - The significance and use of Newton's second law of motion
 - To interpret physical meaning from graphs

Equipment list:

Airtrack (track, shuttle, spring/pulley assembly, blower), string, slotted weights, 5g mass hanger, 2 photogates w/ stands, computer interface

Apparatus:



Theory:

Newton's 2nd law of motion – Newton's Laws of motion have been a long standing standard for examining mechanics in classical situations. The second law in particular is of great use and importance. You probably recognize this law in the form $F=ma$ (force is the product of mass times acceleration), but Newton himself stated "A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed (Philosophiae Naturalis Principia Mathematica)." Mathematically, this law is generally expressed as force being the time derivative of momentum. But then where does this $F=ma$ come from? To answer that, we start with the momentum formulation,

$$F = \frac{dP}{dt} \cong \frac{\Delta P}{\Delta t}$$

Where P is momentum and t is time. We can then substitute mass times velocity for momentum (from $P=mv$).

$$F = \frac{d(mv)}{dt} \cong \frac{\Delta(mv)}{\Delta t}$$

Then, if we assume that mass does not change in this situation, we can separate the mass term.

$$F = m \frac{dv}{dt} \cong m \frac{\Delta v}{\Delta t}$$

And you should be familiar by now that the time derivative (or change in velocity over a time interval) is equivalent to acceleration, which gives the familiar $F=ma$. It warrants saying, though, that one should be a little careful with this equation... it is only valid for “special cases”. In the steps above, there are two major problems, assumptions that are not always valid.

First, momentum is not always mass times velocity. For example, a charged particle moving through an electric field, or objects in relativistic situations both have different definitions of momentum. But in “classical mechanics,” $p=mv$ is correct; this is the same as “everyday events” that you can imagine happening around you. Second, mass is not always unchanged. Imagine a rocket engine. It is propelled forward by burning the fuel and forcing the exhaust out of a nozzle at very high speed, which causes acceleration in the opposite direction. In this case, as the fuel is burned and converted to exhaust, the rocket loses mass. That said, $F=ma$ is fine to use as long as the two types of situations above are not a factor (it works great for the scope of this course), but if one of those two situations comes into the picture (most often it is the changing mass), a new approach has to be taken.

In SI units, mass is expressed in kilograms [Kg], acceleration in meters per second, squared [m/s^2], and force is in Newtons [N], which is equivalent to [$Kg*m/s^2$].

Theoretical acceleration – in this experiment, you will attempt to verify Newton’s second law by using a known force to accelerate a “frictionless” object. You will then compare the actual acceleration of that object to the theoretically predicted acceleration.

A mass hanging from a string over a pulley will act as the “accelerating mass,” applying a force to accelerate the object being observed. The force this accelerating mass (m_a) applies can be determined by multiplying the mass by the acceleration of gravity ($g=9.81m/s^2$), $F_a=m_a g$. The accelerating force is then responsible for accelerating not only the object in question, but the accelerating mass as well (the two masses comprise a single system for this part of the calculation). The acceleration, then, can be calculated from the equation,

$$a = \frac{F_a}{m + m_a}$$

Or substituting for F_a ,

$$a = \frac{m_a g}{m + m_a} \quad (1)$$

Where m_a is the accelerating mass and m is the shuttle (or “object”) mass. The reason both masses (glider and hanger) appear in the denominator is because both objects are accelerating due to the applied force of gravity, and thus both must be accounted for in the mass term of $F=ma$.

Experimental acceleration – While the theoretical acceleration is calculated from Newton’s second law, the experimental acceleration is calculated from the familiar $a=\Delta v/\Delta t$, which makes a discussion of the experimental timing scheme crucial.

There are three regions of time measurement. The first time, t_1 , is the time it takes for the shuttle to pass through the first photogate, beginning when the front of the shuttle blocks the beam and stopping when the back of the shuttle clears the beam. t_2 is a similar measurement; it is the time it takes for the shuttle to pass through the second photogate. The final time, t_3 , is the time between the other two; it begins when the back of the shuttle clears the first photogate and stops when the shuttle enters the second photogate. It is easier to conceptualize the process if we imagine replacing the shuttle with a point mass halfway

between the front and back ends of the shuttle. Figure 1 shows the location regions of this point mass during each time interval.

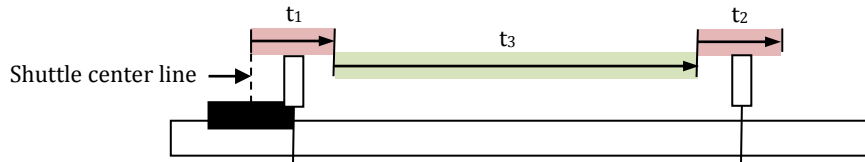


Figure 1: experimental timing scheme depicted as regions of location of a point on the center line of the shuttle.

The average velocity of the shuttle passing through each photogate can be calculated by dividing the length, L , of the shuttle by the time it takes to pass through the photogate, giving

$$v_1 = \frac{L}{t_1} \quad \text{and} \quad v_2 = \frac{L}{t_2} \quad (2)$$

Because the shuttle is accelerating, the calculated velocity is the *average* velocity, not the instantaneous velocity. But... using our imagined point mass, its instantaneous velocity is approximately the same as the shuttle's average velocity. So the acceleration can be calculated as the change in instantaneous velocity of the particle as it passes each photogate divided by the time between each position. The problem though, is there isn't a measured time that corresponds to the distance between the photogates (see figure 1). We can approximate it by using t_3 , and adding half of t_1 and half of t_2 . Thus, the experimental acceleration is calculated with the following equation.

$$a = \frac{v_2 - v_1}{t_3 + \frac{t_2}{2} + \frac{t_1}{2}} \quad (3)$$

Note: It is important that you convert your measurements to SI units before using any of these equations so that your results have the proper units.

Procedure:

You will perform this experiment in two different modes: constant total mass, where the total mass of the system (hanger, shuttle, and the mass added to each) remains constant, but is moved from the shuttle to the hanger between runs; and constant accelerating mass, where the hanger and its added mass remains constant, but the mass added to the shuttle varies (total mass of the system varies in this part).

Initial set-up

1. Set up the airtrack as shown in the apparatus section, with the photogates approximately 50cm apart. Switch on the air pump (black cylinder on the floor). Check that the track is level – the unloaded shuttle, without the string attached, should remain stationary when placed on the track.
2. Plug the first photogate (photogate 1) into channel 1 and photogate 2 into channel 2 on the computer interface. Check that the shuttle does not hit the photogates as it passes through, and the photogates trigger only on the black shuttle (they should not be triggered by the string or the silver clip that connects the string to the shuttle).
3. Measure the length of the glider and record this value as L . Calculate the distance between the photogates by observing the shuttle position when it triggers each photogate (indicated by the red LED), and record this value as D .

4. Weigh the unloaded shuttle, and record this value as m_s .
5. Weigh the unloaded hanger, and record this value as m_h .
6. Attach one end of the string to the clip on the shuttle and the other end to the weight hanger, running the string over the pulley, so the hanger is hanging past the end of the table.
7. Open the *Force_Acceleration_Airtrack* file in the *Physics Lab* folder on the computer desktop.

Part 1 – Constant total mass

8. Add two 5g and four 10g masses to the shuttle. Note: the mass added to the shuttle must always be configured symmetrically (the same amount on each side of the shuttle). The hanger (5g) has no added mass for the first run.
9. Record the total shuttle mass (added mass plus m_s) as m and the total hanger mass (added mass plus m_h) as m_a in the table.
10. One student will hold the shuttle at a starting position on the track, click start on the computer, and then release the shuttle (be careful not to give the shuttle a push).
11. Once the shuttle has cleared photogate 2, another student will catch the shuttle. Be very careful to not trigger the photogates.
12. Record the three times in your table. *Then* click stop on the program – stopping the program before you record the times may result in erroneous data.
13. Reset the shuttle, and move 10g of the mass from the shuttle to the hanger. The total mass of the system should be the same, but distributed differently between the shuttle and hanger. Don't forget to keep the mass symmetric on the shuttle.
14. Repeat steps 8-12 until you have recorded data for at least four different mass distributions.

Part 2 – Constant accelerating mass

15. Adjust the mass on the hanger to one of the previously tested values.
16. Remove all mass from the shuttle.
17. Record m and m_a .
18. Perform a run like you have done before for this mass configuration.
19. Add mass symmetrically to the shuttle and repeat the data collection until you have recorded data for at least five different shuttle masses (do not change the hanger mass).

Data analysis

20. For each run, calculate the total system mass (m_a+m), accelerating force due to gravity acting on m_a ($F_a=m_ag$), and the expected acceleration from eq. (1) and record these values.
21. For each run, calculate v_1 and v_2 from eq. (2) and the experimental acceleration from eq. (3) and record these values.
22. Calculate the %error for each of your experimental acceleration results.

Prelab:

1. If you recorded the height of the hanger during a run, what would the graph of height vs. time look like (what would be the shape created)?
2. If you increased the mass on the hanger in question one, how would the shape of the graph be affected?
3. How could you determine the value of g from a graph of height vs. time... without calculating any velocities?
4. Why must t_1 and t_2 be included in your calculation for the experimental acceleration?

5. It is always important to consider the reasonability of your data and results to catch major problems as early as possible. List the expected order of t_{1-3} in terms of increasing duration. The acceleration must fall between which values (regardless of mass distribution)?

Report:

Constants

Shuttle length, $L =$

Distance between photogates, $D =$

Shuttle mass (unloaded), $m_s =$

Hanger mass (unloaded), $m_h =$

Table 1 - part 1

m	m_a	F_a	t_1	t_2	t_3	v_1	v_2	Expected a	Experimental a	%error

$m+m_a =$

Table 2 - part 2

m	$m+m_a$	t_1	t_2	t_3	v_1	v_2	Expected a	Experimental a	%error

$m_a =$

$F_a =$

Questions:

1. How well do your experimental results conform to your expected results? Were you able to confirm Newton's second law of motion? Be sure to cite your results specifically.
2. Graph the expected and experimental acceleration from part 1 as a function of accelerating force, F_a . What is the relationship between the expected acceleration and F_a (what is the shape of the graph)? Does your experimental acceleration exhibit a similar trend?
3. Graph the expected and experimental acceleration from part 2 as a function of total mass, $m+m_a$. What is the relationship between the expected acceleration and total mass? Does your experimental acceleration exhibit a similar trend?
4. What would you have to do to the data to make the graph in question 3 linear?
5. If you were to mix up your graphs, that is, if you graphed your part 2 data in question 2 and your part 1 data in question 3, what would the graphs look like?